

M.Sc. (Mathematics) - 2nd Semester
(2721)

Paper: MATH-565

Partial Differential Equations and Integral Equations

Time Allowed: 2 hrs.

Max. Marks: 100

Note: There are EIGHT questions of equal marks. Candidates are required to attempt any FOUR questions.

Section - A

- (a) Find the integral surface of the linear PDE $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0, z = 1$.
- (b) Find a complete integral of $p = (z + qy)^2$, using Charpit's method.
- (a) By Jacobi's method, solve the equation $xp + 3yq = 2(z - x^2q^2)$.
- (b) Find the surface which is orthogonal to the one-parameter system $z = cxy(x^2 + y^2)$ and which passes through the hyperbola $x^2 - y^2 = a^2, z = 0$.

Section - B

- (a) By Monge's method, solve the PDE $3r + 4s + t + (rt - s^2) = 1$.
- (b) Obtain the solution, valid when $x, y > 0, xy > 1$, of the differential equation $\frac{\partial^2 z}{\partial x \partial y} = \frac{1}{x + y}$ such that $z = 0, p = 2y / (x + y)$ on the hyperbola $xy = 1$.
- Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$; $0 < x < 1, t > 0$ subject to the conditions $u(0, t) = u(1, t) = 0$, $u(x, 0) = 0$ and $\frac{\partial u(0, t)}{\partial t} = 1$.

Section - C

- Solve the Volterra integral equation $y(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2} y(t) dt$.
- Using the method of successive approximations, solve the Volterra integral equation $y(x) = 1 + x - \int_0^x y(t) dt, y_0(x) = 1$.

Section - D

- Solve the Fredholm's integral equation $y(x) = \cos x + \lambda \int_0^\pi \sin(x-t)y(t) dt$.
- Solve the Fredholm's integral equation, using the method of successive approximations, $y(x) = (x+1)^2 + \int_{-1}^1 (xt + x^2t^2)y(t) dt$.